A note on design of shear reinforcement for circular section

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Introduction

Although circular reinforced concrete (R.C.) sections are commonly used for foundations and columns, it is interesting to note that there is no guideline for design of shear reinforcement in the latest code of practice for reinforced concrete (BD, 2004) used in Hong Kong.

This Technical Note aims to compare two design formulas available for design of shear reinforcement for circular sections.

Rectangular section

For a rectangular section, the commonly accepted formula for design of shear reinforcement is:

$$A_{sv} \geq \frac{b_v\, s_v\, (v - v_c)}{0.87\, f_{yv}} \qquad (1)$$

where b_v = breath of section,
s_v = spacing of links along the member
v = design shear stress across the section
v_c = design concrete shear stress
f_{yv} = characteristic strength of links

The factor 0.87 represents the reciprocal of a material factor applied to the characteristic strength of steel reinforcement based on the 1985 version of the British Code BS8110. For more recent versions of BS8110, the design code becomes less conservative. A smaller material factor is allowed and hence the factor 0.87 in Eq.1 is replaced by 0.95.

In Hong Kong, the code of practice for design of concrete is still largely based on the 1985 version of BS8110. Therefore, we will retain the factor 0.87 in the design equation of Eq.1 and other design formulas presented later in this Note.

For a rectangular section, the effective area A is usually taken as $A = b_v d$, where d is the effective depth of section.

Clarke & Birjandi's method

Shear reinforcements for circular sections are commonly provided in the form of circular links. Clarke & Birjandi (1993) proposed a formula for design of circular links by modifying Eq.1. Before introducing this formula, the following concepts are discussed in Clarke & Birjandi (1993).

a. Effective depth, *d*

Referring to Figure 1(a), the effective depth is defined as "the distance from the extreme compression fibre to the centroid of the tension reinforcement in the opposite half of the member" (Clarke & Birjandi, 1993). If the main reinforcement bars are closely spaced, the centroid C of the tension reinforcement can be approximated by the centroid of a semi-circular arc as shown in Figure 1(b). Using such an approximation, a closed-form, but approximate, solution can be developed for calculating the effective depth as follows:

$$d = r\left(1 + sin\alpha\right) \qquad (2)$$

where $sin\alpha = \dfrac{2\,r_s}{\pi\,r}$

b. Effective area, *A*

For a circular section, the effective area can be taken as the area of shaded zone in Figure 1(a). If the effective depth is approximated by Eq.2, the effective area of a circular section can be taken approximated as:

$$A = r^2\left(\frac{\pi}{2} + \alpha + sin\alpha\ \ cos\alpha\right) \qquad (3)$$

The shear stress is calculated by dividing the shear force by the effective area.

c. Effective breath, *b*

Using Eq.2 and 3, the effective breath of a circular section is defined as:

$$b = \frac{A}{d} = \frac{\frac{\pi}{2} + \alpha + sin\alpha\, cos\alpha}{1 + sin\alpha} \times r \qquad (4)$$

The formula proposed by Clarke & Birjandi (1993) for a circular section is equivalent to replacing the width b_v of a rectangular section in Eq.1 by the effective breath b of the circular section, viz.

$$A_{sv} \geq \frac{b\, s_v \left(v - v_c\right)}{0.87\, f_{yv}} \qquad (5)$$

Eq.5 is based on the material factor used in the 1985 version of BS8110.

Feltham's method

Feltham (2004) adopted the same concept of equivalent depth, equivalent area and equivalent breath of a circular section as Clarke & Birjandi (1993), but argued that one should consider the reduction in effectiveness of a circular link in resisting shear force due to its curvature of the shear reinforcement. For closely spaced main reinforcement and closely spaced circular links, Feltham (2004) developed the following closed-form, but approximate, formula for design for circular links.

$$A_{sv} \geq \frac{2\, r\, k_c\, s_v \left(v - v_c\right)}{0.95\, f_{yv}} \qquad (6)$$

where r = radius of circular section

k_c = a factor depending on the radius r_s of the circle circumscribing the centre main reinforcement bars (see Figure 1). The factor varies over a narrow range close to 1.0.

It should be pointed out that there is typographic error in Feltham's paper. In Eq.8 of the main text of Feltham's paper, the parameter r is missed out. The correct formula presented in Appendix B of Feltham's paper contains the parameter r.

Eq.6 is based on the material factor used in the current version of BS8110. If one adopts the material factor based on the 1985 version of BS8110 (i.e. replacing the factor of 0.95 by 0.87), replaces $2r$ by the diameter D of the circular section and takes the value of k_c as unity for simplicity, Eq.6 is modified as:

$$A_{sv} \geq \frac{D\, s_v\, (v - v_c)}{0.87\, f_{yv}} \tag{7}$$

Feltham's formula is equivalent to replacing b_v in Eq.1 by the diameter of the circular section.

The effective breath b of a circular section is always smaller than the diameter D, Feltham's formula requires a larger area of shear reinforcement to resist the same design shear force as compared with Clarke & Birjandi's formula.

Comparison of two formula

If R is the ratio of required area of shear reinforcement based on Feltham's formula and that based on Clarke & Birjandi's formula, the value of R can be calculated using the formula derived from Eq.4.

$$R = \frac{D}{b} = \frac{2\,(1 + sin\,\alpha)}{\frac{\pi}{2} + \alpha + sin\,\alpha\; cos\,\alpha} \tag{8}$$

Figure 2 shows the variation of R versus the ratio of r_s/r. It can be observed that the Feltham's formula generally requires about 20% more area of shear reinforcement as compared with Clarke & Birjandi's formula.

References

Buildings Department (BD) (2004). *Code of Practice for Structural Use of Concrete*.

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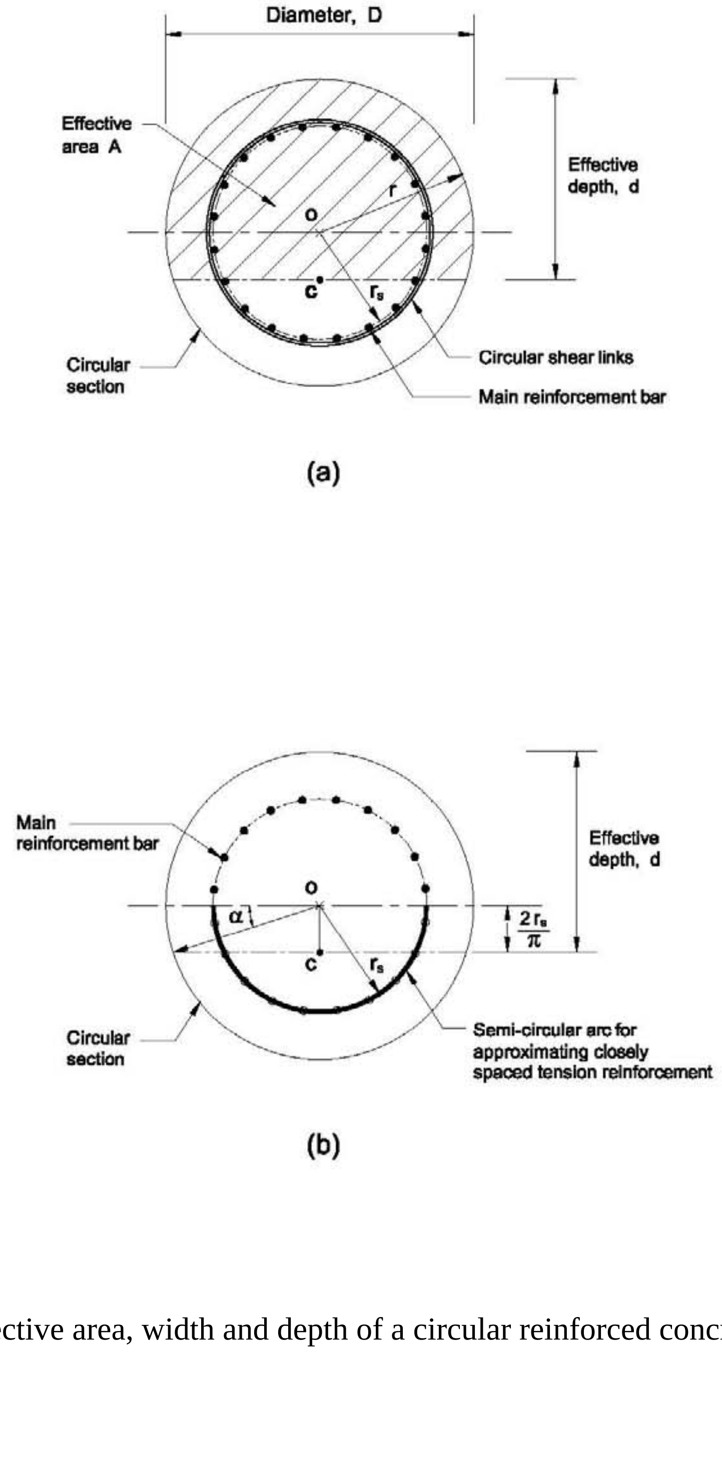

Figure 1 Effective area, width and depth of a circular reinforced concrete section

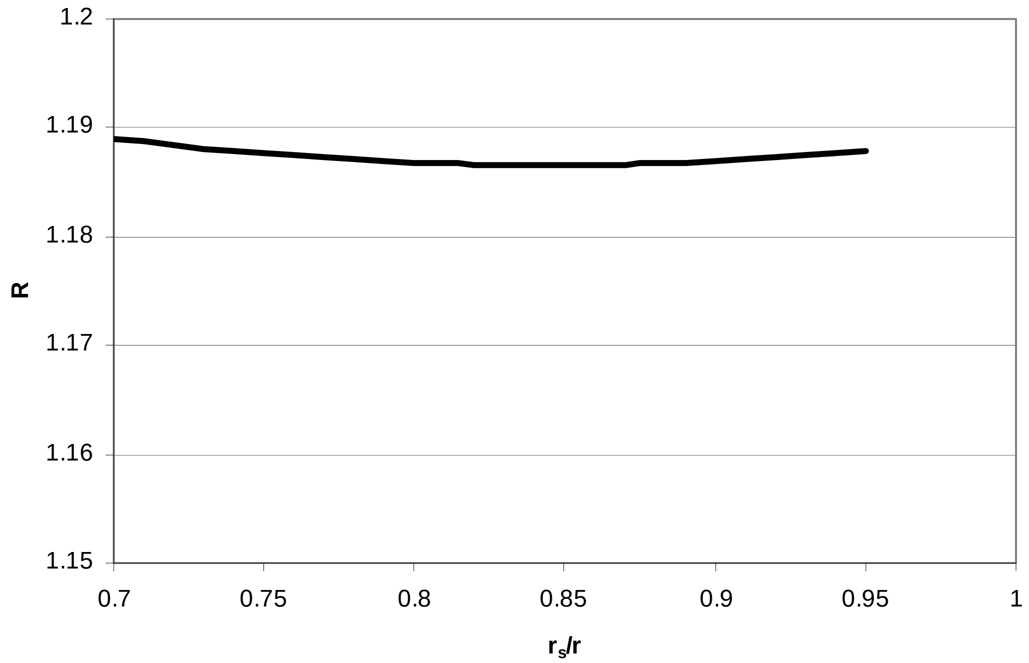

Figure 2 *R* versus r_s/r